Name:_____ Period: ____

Derivative Worksheet

Sections 3.1-5

We've covered 5 sections of derivative formulas; let's recap them one by one. I am fairly certain that I didn't leave you enough room to do all of your work on this sheet; you may need to use a separate piece of paper for your scratch work.

1. The **Power Rule**. There are several ways to precisely state the power rule. One way is to write:

$$\boxed{\frac{d}{dx}\left(x^n\right) = nx^{n-1}}$$

Find two more ways to state the power rule using different derivative notations.

(a) If $f(x) = x^5 + 3x^2$, find $f'(x)$

(b) Find $\frac{d}{dx}\Big(x^{300} + 74x^{243} - 72x^{29} - 6\Big)$.

(c) If $y = \dfrac{mx + b}{d}$ where m, b, d are constants, find $\frac{dy}{dx}$.

2. The **Exponential derivative**. This can be stated as one simple rule, but because the base e delivers such a nice (and surprising) result, it is usually stated on its own. Fill in the blanks:

$$\boxed{\frac{d}{dx}\left(e^x\right) =}$$

$$\boxed{\frac{d}{dx}\left(a^x\right) =}$$

(a) $f(x) = e^x + x^e$, find f'.

(b) $\frac{d}{dx}\left(2^x + 3^x\right) =$

(c) If $z = b^y + y^a + 5y^d$, for constants b, a, d ☺ find $\frac{dz}{dy}$.

3. The **Product rule** and **Quotient rule**. As we'll see at the end of this worksheet, the only one of these that is *absolutely* necessary is the product rule, but I think it's worth knowing both. It is important to know that there is a difference between how you remember them and how you precisely state them. Give preciseness a try:

$$\boxed{\frac{d}{dx}\Big(f(x)\cdot g(x)\Big) =}$$

$$\boxed{\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) =}$$

(a) $f(x) = (x^2+3x+1)(x^4-5x^3+3)$. Find $f'(x)$.

(b) $g(x) = (x^2+3x)\cdot e^x \cdot 5^x$. Find $g'(x)$.

(c) $y = \dfrac{x^5+3x^3-7x}{x\cdot e^x}$

As another point of interest, here is another way to write the product rule using Leibniz's notation. Below, x and y are both functions of t and the *derivative* of z is with respect to t:

$$z = x\cdot y,$$
$$\frac{dz}{dt} = \frac{dx}{dt}\cdot y + x\cdot\frac{dy}{dt}$$

If $z = \dfrac{x}{y}$, write the quotient rule using Leibniz's notation for $\frac{dz}{dt}$.

4. **Trigonometric Derivatives**. We spent a long time proving that the derivative of Sine was Cosine, and similar work would have found the derivative of Cosine too. The rest can be derived from those two using the product and quotient rules. List all 6 trigonometric derivatives below (be sure to use some sort of correct notation, there are many ways to write it):

The two *facts* that I wrote on the board aren't worth memorizing, but they are worth "familarizing" because they do pop up once in a while.

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1} \qquad \text{and} \qquad \boxed{\lim_{x\to 0}\frac{\cos x-1}{x}=0}$$

(a) If $g(x)=\sqrt{x}\cdot\sin x$, find $g'(x)$.

(b) Find the equation of the tangent line to the curve $y=\tan x$ at the point $(\pi/4,1)$.

(c) Find the equation of the tangent line to the curve

$$y=\frac{1}{\sin x+\cos x}$$

at the point $(0,1)$. Once you have finished, confirm your results by graphing the curve and the tangent line on the same screen.

(d) Find all points on the graph of the function $f(x)=2\sin x+\sin^2 x$ at which the tangent line is horizontal. Hint: horizontal tangent lines happen at points where the *derivative is equal to 0*. You should solve for those points.

5. The Grandaddy of all derivative rules, the dreaded **Chain Rule**. If $h(x) = f\big(g(x)\big)$, then

$$\boxed{h'(x) = f'\Big(g(x)\Big) \cdot g'(x)}$$

Or

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

(a) If $f(x) = \csc(x^2)$, find $f'(x)$.

(b) Suppose that f is differentiable on $\mathbb{R}$ (*i.e.* its derivative exists for every real number). Let $F(x) = f(\cos x)$ and $G(x) = \sin(f(x))$. Find expressions for (a) $F'(x)$ and (b) $G'(x)$. Hint: Chain rule. For (a) the inside function is $\cos x$ and the outside function is $f(x)$.

(c) Prove that the Chain Rule and the Product Rule together imply the Quotient Rule:

$$\text{Hint: } \frac{f(x)}{g(x)} = f(x) \cdot [g(x)]^{-1}$$

(d) This is the rule that requires the *most* practice, and you only get it by doing lots of hard problems. Your homework (Due on Wednesday 10/21 to accommodate weekend retreaters) is to do Handout 3-4, problems 1-53 e.o.o. and 61-64 (it's 19 problems, don't freak out). The more you do, the easier they will get for you. Please, Please, Please, stop buy after school and do some problems while in my presence, whether you understand or not. I can help clear up fuzzies.

6. We only have two "derivative rules" left (logs and hyperbolic trig functions) and one derivative technique (implicit differentiation) then we move on to applications (where we'll prove several facts used in AP physics, 'cause we're better).